VARIATIONAL APPROACH TO THE ANALYSIS OF THE EFFECT OF A DISTRIBUTION GRID ON THE QUALITY OF FLUIDIZATION

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UDC 66.096.5

The article presents a theoretical analysis of the flow distribution in an apparatus with a fluidized bed on the assumption of minimum energy loss in the movement of the fluidizing agent through a distribution grid and the bed. The conditions of uniform fluidization were found.

To ensure normal operation of apparatus with a fluidized bed, characterized by uniform distribution of the fluidizing agent over the cross section of the bed, absence of stagnant zones and through channels, it is of great importance to choose correctly the parameters of the distribution grid [1]. A review of a large number of works connected with the investigation of the effect of distribution grids on the quality of the fluidization and the selection of their optimum parameters is contained in the monographs [1, 2]. An analysis of past investigations leads to the conclusion that the methods of selecting distribution grids for apparatus with a fluidized bed suggested in the literature are not always sufficiently reliable.

The present work constitutes an attempt to approach the problem of selecting a distribution grid by using the variational principle of mechanics which was successfully used in a number of investigations for the analysis of fairly complex phenomena characterizing the processes of fluidization, and specifically for describing a nonuniform fluidization regime [3].

On the basis of the variational principle it may be assumed that the movement of the fluidization agent through a grid and bed is accompanied by such a redistribution of the speeds over the cross section of the apparatus which ensures minimum energy losses of the fluidizing agent. We assume further that the cross section of the apparatus can be arbitrarily divided into two zones: zone 1 with cross section S_1, where the speed $w_1 > \bar{w}$, so that the bed in this zone is in the fluidized state with porosity ε_1, and zone 2, where the speed $w_2 < w_0$, as a result of which the bed in zone 2 is immobile and has porosity ε_0. In addition to that we assume that in consequence of the liquid state of the bed, the height of the bed in zones 1 and 2 is equal and amounts to

$$H = H_0 \frac{1-\varepsilon_0}{1-\bar{\varepsilon}}. \quad (1)$$

The total energy loss of the fluidizing agent upon passage through the grid and bed for the adopted model of flow distribution can be found from the equation

$$\Delta E = \Delta P_1 w_1 S_1 + \Delta P_2 w_2 S_2. \quad (2)$$

For the subsequent analysis we introduce the dimensionless magnitudes

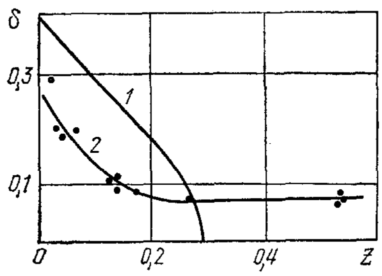

Fig. 1. Dependence of δ on Z with u = 0.1.for glass spheres: 1) theoretical; 2) experimental curve [5].

All-Union Research Institute of Organic Synthesis, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 39, No. 4, pp. 681-686, October, 1980. Original article submitted September 6, 1979.

 0022-0841/80/3904-1108$07.50

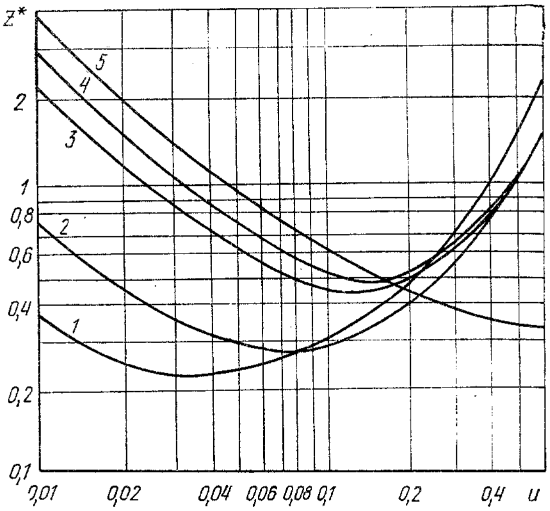

Fig. 2. Dependence of Z* on u for different Archimedean numbers: 1) 10^2; 2) 10^4; 3) 10^6; 4) 10^8; 5) curve obtained by Zabrodsky's equation [2] for Ar = 10^6.

$$\Delta\mathscr{E} = \frac{\Delta E}{\Delta\tilde{P}_{fl}\,\bar{w}S}; \quad Z = \frac{\Delta\tilde{P}_g}{\Delta\tilde{P}_{fl}};$$

$$s_1 = \frac{S_1}{S}; \quad s_2 = \frac{S_2}{S}; \quad \omega_1 = \frac{w_1}{\bar{w}}; \quad \omega_0 = \frac{w_0}{\bar{w}}; \quad \psi = \frac{w_2}{w_0}, \tag{3}$$

where the hydraulic resistances of the grid $\Delta\tilde{P}_g$ and of the fluidized bed $\Delta\tilde{P}_{fl}$ in uniform fluidization are determined by the relationships

$$\Delta\tilde{P}_g = \frac{\xi\rho\bar{w}^2}{2F^2}, \tag{4}$$

$$\Delta\tilde{P}_{fl} = (\rho_s - \rho)(1 - \varepsilon_0)gH_0. \tag{5}$$

The use of the dimensionless variables makes it possible to write the general balance equations and expressions for energy losses of the fluidizing agent in the following dimensionless form:

$$s_1 + s_2 = 1, \quad \omega_1 s_1 + \psi\omega_0 s_2 = 1, \quad \varepsilon_1 s_1 + \varepsilon_0 s_2 = \bar{\varepsilon}, \tag{6}$$

$$\Delta\mathscr{E} = \left(Z\omega_1^2 + \frac{1-\varepsilon_1}{1-\bar{\varepsilon}}\right)\omega_1 s_1 + \left(Z\psi^2\omega_0^2 + \psi\frac{1+\eta\psi}{1+\eta}\,\frac{1-\varepsilon_0}{1-\bar{\varepsilon}}\right)\psi\omega_0 s_2, \tag{7}$$

where the parameter $\eta = K_1 Re_0/12K(1 - \varepsilon_0)$, and the Reynolds number is $Re_0 = w_0 d_a/\nu$.

For calculating the porosity of the fluidized bed we may use Todes' [4] relationship for uniform fluidization regime (according to which $\varepsilon = \varepsilon(Ar, Re)$, where the Archimedean number is $Ar = \frac{gd_a^3}{\nu^2}\frac{\rho_s - \rho}{\rho}$) or the corresponding relationship for nonuniform fluidization, e.g., the equation obtained in [3].

If we use relationship (6) and the relationship for calculating ε, we can eliminate the quantities s_1, ω_1, ε_1, and $\bar{\varepsilon}$ from Eq. (7) and represent $\Delta\mathscr{E}$ as a function of only two unknowns, viz., s_2 and ψ. The obtained equation describing the function $\Delta\mathscr{E}(s_2, \psi)$ is very cumbersome, and it is therefore not given here. In accordance with the variational principle, the magnitudes s_2 and ψ can be found by minimizing the function $\Delta\mathscr{E}(s_2, \psi)$, however, since the function is very nonlinear with respect to the sought variables, their accurate analytical determination is at present impossible.

To verify the correctness of the suggested model, the minimization of the function $\Delta\mathscr{E}(s_2, \psi)$ was carried out numerically with respect to the experimental data of [5], where the results of the measurement of the relative dispersion of the concentration of the solid phase δ near the grid, obtained with the aid of a capacitive sensor, are given. In accordance with the flow-distribution model adopted in the present work and with the assumption that the zones 1 and 2 are fixed in space and may be viewed as random variables with probabilities of being at the given point over the cross section of the apparatus equal to s_1 and s_2, respectively, for the values δ we can obtain the ratio

$$\delta = \frac{\sqrt{(\bar{\varepsilon}-\varepsilon_1)^2 s_1 + (\bar{\varepsilon}-\varepsilon_0)^2 s_2}}{1-\bar{\varepsilon}}.$$

As an example, Fig. 1 presents the results of calculating the dependence of δ on Z with the reduced speed of the fluidizing agent $u = 0.1$ for glass spheres. In the calculations, the porosity of the fluidized bed was found by Todes' [4] equation, and for ε_0 the value 0.476 [1] was adopted which corresponds to the porosity of a loosened fluidized bed at the speed of the onset of fluidization. It can be seen from Fig. 1 that the theoretical curve (for $Z < Z^*$) agrees qualitatively well with the experiment; however, the theoretical values of the relative dispersion of the concentration of the solid phase are approximately twice as large as the corresponding experimental data. The agreement may be considered satisfactory in view of the fact that, as the authors of [5] note, δ increases with increasing distance from the grid, whereas the data on the magnitude of δ, presented in [5], are derived from measurements near the grid.

Since the determination of the grid parameters ensuring uniform fluidization over the entire cross section of the apparatus is of practical importance, it is interesting to examine the function $\Delta\mathscr{E}(s_2, \psi)$ with extremely small values of s_2. For this purpose we expand the function $\Delta\mathscr{E}(s_2, \psi)$ into a series according to the magnitude of s_2, and confining ourselves to two terms of the expansion, we obtain

$$\Delta\mathscr{E} = Z + 1 + s_2\left[2Z - \beta - \psi\omega_0(3Z+1) + Z\psi^3\omega_0^3 + \alpha\psi^2\omega_0\frac{1+\eta\psi}{1+\eta}\right], \quad (8)$$

where, for the sake of the brevity, we denote

$$\alpha = \frac{1-\varepsilon_0}{1-\tilde{\varepsilon}}; \quad \beta = \frac{\tilde{\varepsilon}-\varepsilon_0}{1-\tilde{\varepsilon}}. \quad (9)$$

The minimum of the function $\Delta\mathscr{E}(s_2, \psi)$ corresponds specifically to the condition $\partial\Delta\mathscr{E}/\partial\psi = 0$, which enables us to write the relationship

$$3\left(Z\omega_0^2 + \alpha\frac{\eta}{1+\eta}\right)\psi^2 + 2\alpha\frac{\psi}{1+\eta} - 3Z - 1 = 0. \quad (10)$$

Since according to the physical essence of the model, $s_2 \geq 0$, it follows from Eq. (8) that the minimum of the function $\Delta\mathscr{E}(s_2, \psi)$ is attained with $s_2 = 0$ (which corresponds to the condition of uniform fluidization) only if the following inequality applies:

$$2Z - \beta - \psi\omega_0(3Z+1) + Z\psi^3\omega_0^3 + \alpha\psi^2\omega_0\frac{1+\eta\psi}{1+\eta} \geqslant 0. \quad (11)$$

In consequence of the considerable nonlinearity of the system of equations (10) and (11) with respect to the sought variables ψ and Z, the smallest permissible value Z^*, corresponding to the equality sign in relationship (11), was found numerically. Here, as before, the value $\varepsilon_0 = 0.476$ and Todes' equation [4] were used. The results of the calculations are presented graphically in Fig. 2 in the form of the dependence of Z^* on u in logarithmic coordinates for different Archimedean numbers. The established correlation $Z^* = Z^*(u, Ar)$ may be used for practical purposes. For approximate calculations a relationship may be recommended which was obtained as a result of the analytical solution of the system of equations (10) and (11) with the condition $\eta \gg 1$. This solution has the form

$$Z^* = \frac{B + \sqrt{B^2 - 4AC}}{2A}, \quad (12)$$

where

$$A = \alpha(W^2 - 1); \quad B = \alpha\beta W^2 + \frac{1}{3} - \frac{\beta^2}{4}; \quad C = \frac{\alpha\beta^2}{4}W^2 - \frac{1}{27} \quad (13)$$

and $W \equiv \bar{w}/w_0 = 1/\omega_0$ is the fluidization number.

The limited amount of direct experimental data on Z^* makes it difficult to compare the obtained results with experimental results. We want to point out, however, that the calculation is in good agreement with the data of [5], in accordance with which the value of Z^* can be indirectly determined from the graph of the dependence of δ on Z as that value of Z above which the grid parameters do not affect the magnitude of δ (see Fig. 1). To permit comparison of the results of the present analysis with Zabrodsky's equation [2], Fig. 2 shows a curve plotted according to this equation for the number $Ar = 10^6$ using the coefficient $k = 0.5$, which is contained in the examined equation and which was recommended by Zabrodsky [2] on the basis of the processing of the experimental data. It follows from Fig. 2 that for small values of u, the dependences of Z^* on u, calculated in the present work and according to Zabrodsky's equation, are in good qualitative agreement with each other. In view of the approximate nature of Zabrodsky's equation the results of the comparison may be considered fully satisfactory. Since by definition $Z = W\tilde{P}_g/\Delta\tilde{P}_{fl}$, the value of Z^* makes it possible to choose a grid that has the required hydraulic resistance and ensures uniform fluidization.

NOTATION

S, cross-sectional area of the apparatus or of the corresponding zone; w, speed of the fluidizing agent; ε, porosity of the bed; H, height of the bed; ΔE, energy loss of the fluidizing agent; ΔP, hydraulic resistance; $\Delta\mathscr{E}$, Z, s_1, s_2, ω_1, ω_0, and ψ, dimensionless magnitudes determined by Eq. (3); ξ, resistance coefficient of the grid; ρ, density of the fluidizing agent; ρ_s, density of the particles of the solid phase; F, clear cross section of the grid; g, acceleration of gravity; $\eta = K_i Re_0/12K(1 - \varepsilon_0)$; K, Coseni–Karman constant; K_i, inertial component of the coefficient of hydraulic resistance of the fixed granular bed; $Re = wd_a/\nu$, the Reynolds number for the particles; d_a, particle diameter taking the shape factor into account; ν, kinematic viscosity of the fluidizing agent; $Ar = gd_a^3(\rho_s - \rho)/\nu^2\rho$, Archimedean number; δ, relative dispersion of the concentration of the solid phase; $u = (\bar{w} - w_0)/(w_* - w_0)$, reduced speed of the fluidizing agent; w_*, swirling speed of the particles; Z^*, minimum permissible value of Z ensuring conditions of uniform fluidization; α and β, dimensionless quantities determined by Eqs. (9); A, B, and C, dimensionless quantities determined by Eqs. (13); W, fluidization number; k, coefficient in Zabrodsky's equation [2]. Subscripts: 1) zone 1; 2) zone 2; 0) at the speed of the onset of fluidization; fl) fluidized bed; g) grid. A bar above a quantity denotes its mean value, a tilde denotes uniform fluidization.

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